Midterm 2 Review

Math 150 — Calculus I — Spring 2023

Full Name: _____

Red ID:

Show all of your work.

- 1. Consider the function $F(t) = t^3 5t + 1$
 - (a) Find the derivative of the function using the definition of derivative.

(b) Show that F(t) has two horizontal tangent lines.

- 2. Compute the derivative f'(x) for the following functions
 - (a) $y = e^{\cos x} + \cos(e^x)$

(b) $y = \sin(\tan\sqrt{1+x^3})$

(c)
$$y = \sin\left(\frac{e^x}{1+e^x}\right)$$

3. If $x^2 + xy + y^3 = 1$, find the value of y'' at the point where x = 1

- 4. Suppose $f(x) = x + 2\cos x$
 - (a) Compute f'(x)

(b) Find a point in the interval $(0, \pi/2)$ at which curve y = f(x) has the horizontal tangent.

5. Use logarithmic differentiation to find the derivative of the function

$$y = \frac{e^{-3x}\sqrt{x+1}}{(x+3)^7}$$

- 6. A bacteria culture contains 200 cells initially and grows at a rate proportional to its size. After half an hour the population has increased to 360 cells.
 - (a) Find the number of cells after t hours.

(b) Find the number of cells after 4 hours.

(c) Find the rate of growth after 4 hours.

(d) When will the population reach 10,000?

- 7. If the equation of motion of a particle is given by $s(t) = A\cos(\omega t + \delta)$, the particle is said to undergo simple harmonic motion. Here, A, ω , and δ are constants.
 - (a) Find the velocity of the particle at time t.

(b) Find the acceleration of the particle at time t.

(c) When is the acceleration 0?

8. A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3s later?